

2413-110 Discrete Mathematic

Quiz 3 27/Mar/2009 11:05 – 12:30 (85 minutes)

Name _____ Sect _____ ID _____

Notes

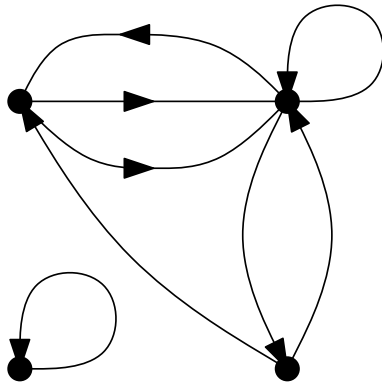
- This paper has 2 question pages and 3 blank answer sheets. **Write down your name and ID on top of every page.**
- Write down your answer **only** in the blank sheet.
- No calculator, closed book, and don't bring any paper in.
- Cheating will not be tolerated.
- Here are some useful theorems and corollaries

Euler's Formula: Let G be a connected planar simple graph with e edges and v vertices.

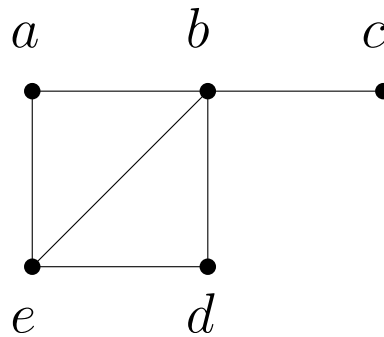
Let r be the number of regions in a planar representation of G . Then $r = e - v + 2$

Corollary 1 of Euler's Formula: If G is a connected planar simple graph with e edges and v vertices where $v \geq 3$, then $e \leq 3v - 6$

Corollary 2 of Euler's Formula: If G is a connected planar simple graph, then G has a vertex of degree not exceeding five.



(a) The graph for problem 1



(b) The graph for problem 4

1. (1 point) The **converse** of a directed graph $G = (V, E)$ is the directed graph (V, F) where $(u, v) \in F$ if and only if $(v, u) \in E$. Draw the converse of the graph in Figure (a).
2. (1 point) What value of n greater than 2 makes C_n bipartite? Why?
3. (1 point) A diagnostic message can be sent out over a computer network to perform tests over all links and in all devices.

- (a) What sort of paths should be used to test all links?
- (b) What sort of paths should be used to test all devices?
4. (2 points) An **orientation** of an undirected simple graph is an assignment of directions to its edges so that the resulting directed graph is strongly connected. When an orientation of an undirected graph exists, this graph is called **orientable**. Determine whether the graph in Figure (b) is orientable.
5. (2 points) What is wrong with the following “proof” using mathematical induction of the statement that every tree with n vertices has a path of length $n - 1$.
- Basis step: Every tree with one vertex clearly has a path of length 0.
 - Inductive step: Assume that a tree with n vertices has a path of length $n - 1$, which has u as its terminal vertex. Add a vertex v and the edge from u to v . The resulting tree has $n + 1$ vertices and has a path of length n . This completes the induction step.
- (hint: it’s a very big fault.)
6. (2 points) A **caterpillar** is a tree that contains a simple path such that every vertex not contained in this path is adjacent to a vertex in the path. Draw all nonisomorphic caterpillars with 6 vertices.
7. (3 points) Show that a full m -ary balanced tree of height h has more than m^{h-1} leaves.
8. (3 points) The **thickness** of a simple graph G is the smallest number of planar subgraphs of G that have G as their union. For example, $K_{3,3}$ has 2 as its thickness. Show that if G is a connected simple graph with v vertices and e edges, then the thickness of G is at least $\lceil e/(3v - 6) \rceil$.

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